An Improved FDTD Model for the Feeding Gap of a Thin-Wire Antenna

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***Abstract*— In calculations using the finite-difference time-domain (FDTD) method, the feeding gap of a thin-wire antenna is often modeled by a so-called "one-cell gap" which lets the feeding gap to be one interval of Yee's lattice. This is often inconsistent with the actual situation and it causes error in FDTD calculation results. This letter shows that the error due to the one-cell gap model is strongly dependent on the cell size, and we present an improved FDTD model which assumes an infinitesimally narrow feeding gap. We show that the antenna input impedance calculated with the new gap model is barely affected by the cell size and agrees well with the method of moments (MoM) calculation results for an infinitesimal gap. Furthermore, we clarify the dependence of error of a one-cell gap on the cell size on the basis of the proposed model.**

***Index Terms*—Antenna input impedance, FDTD method, feeding gap, thin-wire antenna.**

I. Introduction

The finite-difference time-domain (FDTD) method has been applied to various electromagnetic analyses. In particular, analysis of the interactions between a cellular phone and a human head is one of the most effective applications of FDTD method as it includes the computation of internal electromagnetic fields within the human head [1]–[3], i.e., a lossy dielectric body, which is difficult to treat using the method of moments (MoM).

A wire antenna is a common radiating structure for portable communication devices. The feeding gap of a wire antenna is often modeled for FDTD calculation by the so-called "delta gap model" [4] or "one-cell gap model" [5], which lets the feeding gap to be one spatial interval of Yee's lattice [6]. However, the actual feeding gap is usually smaller than the cell size. The conventional one-cell gap model can therefore cause cell-size-dependent error in FDTD computation.

In this letter the characteristics of error due to the one-cell gap model are investigated through a comparison between the calculated antenna input impedance of a thin-wire half-wavelength dipole obtained by the one-cell gap FDTD model and that obtained by MoM where an infinitesimally narrow gap is assumed. We then develop an improved feeding gap model for FDTD calculation which assumes an infinitesimal feeding gap instead of a one-cell gap. Furthermore, we show that the proposed model clarifies how the one-cell gap model yields the cell-size-dependent error when it is applied to a narrow feeding gap.

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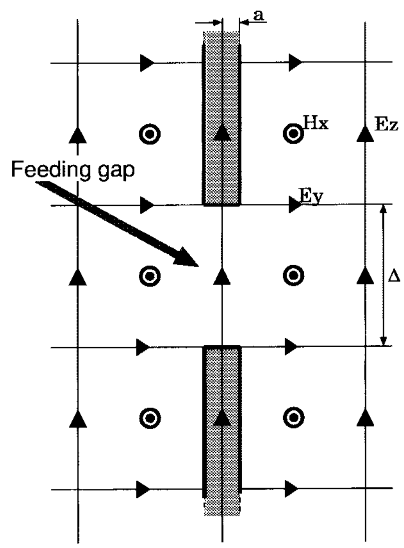

Fig. 1. One-cell gap model of a thin-wire dipole antenna.

II. One-Cell Gap Model

Fig. 1 illustrates a one-cell gap model, or delta gap model, which has been used in various applications [1]–[4]. The feeding source is given by the E-field in the air gap corresponding to one-space interval of Yee's lattice

$$E(\text{gap}) = -V/\Delta \tag{1}$$

where V is the input voltage as a function of time and Δ is the interval of Yee's lattice. Substituting (1) in the ordinary FDTD formula [7], the H-fields around the gap are given by

$$\begin{aligned} H_x^{n+1/2}&(i, j+1/2, k+1/2) \\ &= H_x^{n-1/2}(i, j+1/2, k+1/2) \\ &\quad + \frac{\Delta_t}{\mu_0 \Delta}\big[\{E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k)\} \\ &\qquad - \{E_z^n(i, j+1, k+1/2) + V^n/\Delta\}\big] \end{aligned} \tag{2}$$

where the gap is located at $(i, j, k+1/2)$. Since the E-field in the gap and H-fields surrounding the gap are dependent on the lattice interval Δ as described in (1) and (2), the one-cell gap model can cause cell-size-dependent error in FDTD calculation when the actual gap length is different from the lattice interval.

To investigate the characteristics of the error, the antenna input impedance of a thin-wire dipole antenna in free space

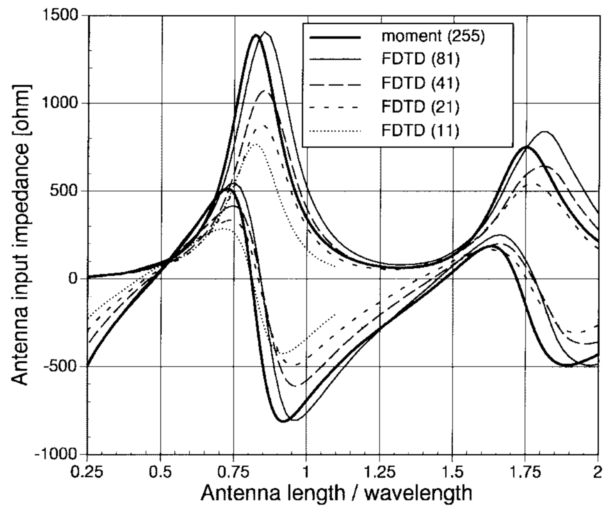

Fig. 2. Input impedance of the thin-wire dipole with the one-cell gap. The antenna radius a is $\ell/500$.

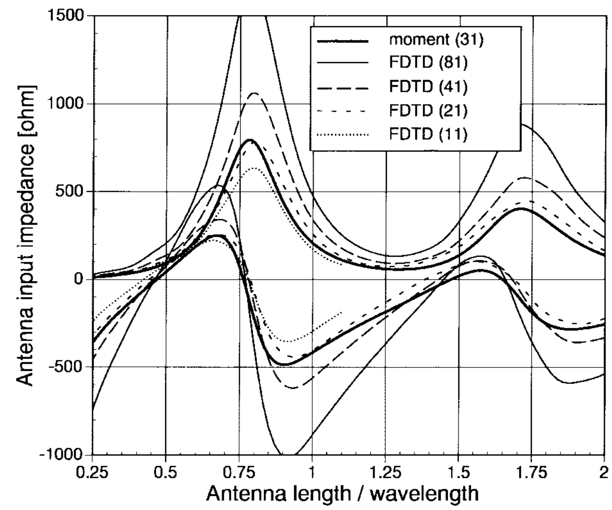

Fig. 3. Input impedance of the thin-wire dipole with the one-cell gap. The antenna radius a is $\ell/150$.

calculated using the one-cell gap FDTD model were compared with those obtained by MoM [8], which assumes an infinitesimal gap. The results are shown in Figs. 2 and 3.

The FDTD calculation parameters are as follows:

- antenna length/wave length: $\ell/\lambda = 0.25 \sim 2.0$;
- antenna radius:

$$a = \ell/500 \text{ in Fig. 2}$$
$$a = \ell/150 \text{ in Fig. 3;}$$

- cell size: $\Delta = \ell/11, \ell/21, \ell/41, \ell/81$;
- calculation region: $2\ell \times 2\ell \times 2\ell$;
- boundary conditions: 2nd approximations of Mur's absorbing boundary condition [9];
- the subcell method [10] is applied to model a smaller radius for the thin-wire antenna than the cell size.

In MoM calculation, the Galerkin's Method with piecewise sinusoidal functions was employed and the antenna wire was divided into 255 and 31 segments for $a = \ell/500$ and $\ell/150$, respectively.

For $a = \ell/500$ (Fig. 2), the calculation using smaller cells seemed to converge to the solution obtained by MoM. For $a = \ell/150$ (Fig. 3), however, this was not true; calculation with the relatively larger cell size, $\Delta = \ell/21$, produced values closer to the MoM results than calculations with smaller cells.

These results indicate that the one-cell gap is not a good model for a narrow gap and error is dependent on the cell size. They also indicated that the expectation that one-cell gap FDTD calculation would converge to MoM calculation for an infinitesimal gap as the cell size decreased is not always true.

III. Infinitesimal Gap Model

Instead of the one-cell gap, we considered a new gap model for FDTD calculation in which the antenna gap is infinitesimally small, as Fig. 4 shows. Here, the E-field in the infinitesimal gap can be represented as

$$E(\text{gap}) = -V\delta(z) \quad (3)$$

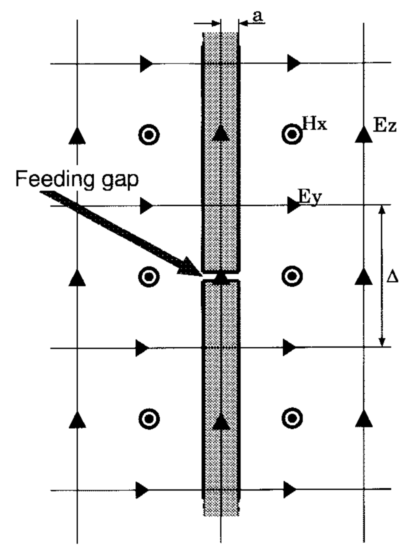

Fig. 4. Infinitesimal gap model of a thin-wire dipole antenna.

where δ is the impulse function of z and the origin is at the center in the gap.

Since the H-fields around the gap are predominantly induced by the antenna current flowing near the gap, the H-fields are assumed to have $1/r$ dependence, where r is the distance from the antenna axis. We can then apply the subcell method [10] to H-field calculations around the gap and obtain the new FDTD formula as follows:

$$\begin{aligned} H_x^{n+1/2}&(i, j+1/2, k+1/2) \\ &= H_x^{n-1/2}(i, j+1/2, k+1/2) \\ &\quad + \frac{\Delta_t}{\mu_0 \Delta}\Bigg[\left\{E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k)\right\} \\ &\quad - \frac{2}{\ln(\Delta/a)}\left\{E_z^n(i, j+1, k+1/2) + V^n/\Delta\right\}\Bigg]. \end{aligned} \quad (4)$$

The antenna input impedance obtained by FDTD calculations using this infinitesimal gap model are shown in Figs. 5

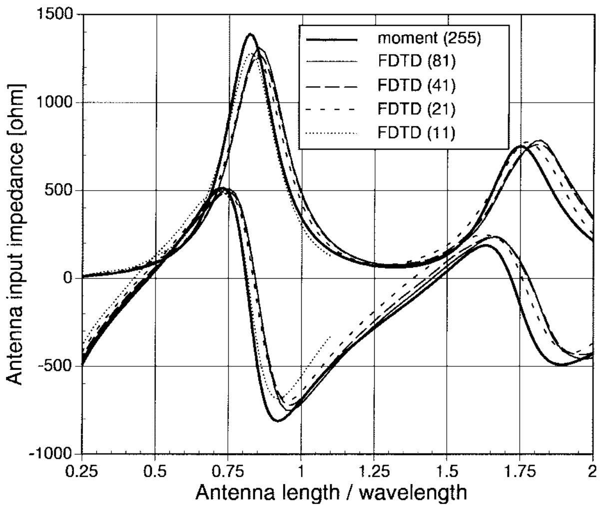

Fig. 5. Input impedance of the thin-wire dipole with the infinitesimal gap. The antenna radius a is $\ell/500$.

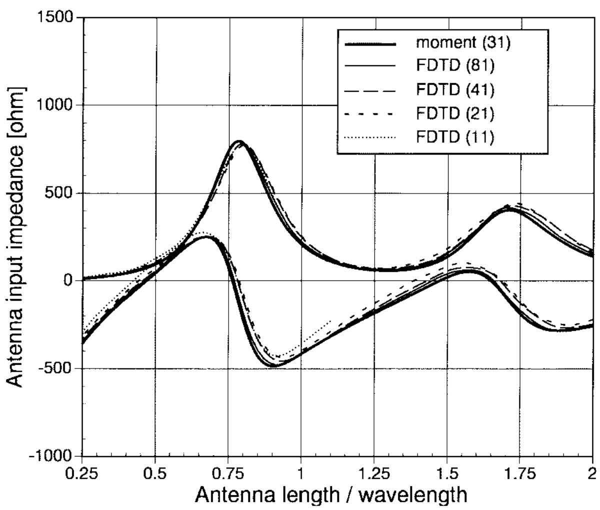

Fig. 6. Input impedance of the thin-wire dipole with the infinitesimal gap. The antenna radius a is $\ell/150$.

and 6. The calculation parameters are the same as those in Figs. 2 and 3.

The FDTD calculations with the infinitesimal gap model agree well with the MoM results and are barely dependent on the cell size in contrast to the one-cell gap model.

IV. Discussion

It is interesting that even though the one-cell gap model assumes a finite gap length equal to the lattice interval, the one-cell gap FDTD with smaller cell size does not always give a better approximation for the infinitesimal gap, while the relatively coarse cell size gives better results in Fig. 3. This can be explained as follows.

The difference between (2) and (4) is in the coefficient of the third term on the right, i.e., 1 and $2/\ln(\Delta/a)$. This suggests that the effective radius $a_{\rm eff}$ of the antenna is implicitly assumed in the one-cell gap model, satisfying $2/\ln(\Delta/a_{\rm eff}) = 1$, that is,

$$a_{\rm eff} = \Delta/e^2 \simeq 0.135\Delta. \quad (5)$$

TABLE I
Effective Radius $a_{\rm eff}$ for Calculations of Fig. 2 ($a = \ell/500$) and Fig. 3 ($a = \ell/150$)

Δ	$\ell/11$	$\ell/21$	$\ell/41$	$\ell/81$
$a_{\rm eff}$	$\ell/81.3$	$\ell/155.$	$\ell/303.$	$\ell/599.$

Table I shows the effective radius for the calculations in Figs. 2 and 3. When the effective radius $a_{\rm eff}$ approximates the actual radius a, FDTD calculation with one-cell gap formulation approximates that with infinitesimal gap formulation. Hence the results of $\Delta = \ell/81$ for $a = \ell/500$ and $\Delta = \ell/21$ for $a = \ell/150$ agree well with the MoM calculation results for infinitesimal gap.

V. Conclusion

We showed that the one-cell gap model for a thin-wire antenna can cause error in calculated antenna input impedance, and this error is strongly dependent on the FDTD cell size.

We also presented an improved feeding gap model for a narrow gap. The results of FDTD calculation using this model agreed well with MoM calculation assuming an infinitesimal gap, and they were barely affected by cell size.

The dependence of error on cell size was discussed based on formulations for a one-cell gap and infinitesimal gap models. We showed that a one-cell gap model for a narrow gap agreed with the infinitesimal gap model only if the cell size was chosen so that the effective radius of the antenna approximated the actual radius.

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